A CRASH COURSE ON MOTIVES

Abstract. The theory of motives is a tool to study arithmetic and geometric questions of algebraic equations in a universal way. In principle, the motive associated to algebraic equation contains all the (linear) information one needs to know from arithmetic and geometric perspective. The idea first originated during the time when the Grothendieck school was making impressive progress in proving Weil conjectures. In fact, the Weil conjectures (now theorems) are very concrete examples where geometry and arithmetic decides each others fate and therefore it is no surprise that the dream (Grothendieck's dream) of a universal way of doing geometry and arithmetic together originated during this time. After having a philosophical beginning the idea gained momentum in 80's when Deligne and Beilinson extended the idea of Grothendieck motives to a conjectural category of mixed motives to incorporate higher K-theory and later Bloch and Beilinson gave several reasons to believe in the existence of mixed motives. Then, during nineties came a great thrust in this area when a proper candidate for the derived category of mixed motives were constructed by Voevodsky (also Levine and Hanamura). The construction of Voevodsky, a beautiful and natural mixture of algebraic geometry and algebraic topology, led to solutions of Milnor conjecture and Bloch-Kato conjecture. Not only that, several other arithmetic questions related to transcendence theory were answered in subsequent decades using this theory.

My goal in this mini course will be to first introduce the philosophy of motives, a la Grothendieck. Then we will see a concrete construction of the category of homological motives, which is (modulo standard conjectures) the theory of motives for smooth projective varieties. Then we will be introduced to the philosophy of mixed motives, a la Deligne-Beilinson. We will see a conjectural candidate for this category, called Nori motives. After this we will see the construction of Voevodsky motives, which is the candidate for the derived category of mixed motives. Along the way we will describe several results which have strengthened the role of motives in arithmetic questions.

I will assume familiarity with basic concepts of algebraic geometry, Grothendieck topology, sheaf cohomology, basic algebraic topology.

References

[1] Mazza, Carlo; Voevodsky, Vladimir; Weibel, Charles (2006), Lecture Notes on Motivic Cohomology, American Mathematical Society.

[2] A, J , Scholl; Classical motives, Motives, Seattle 1991, ed. U. Jannsen, S. Kleiman, J-P. Serre. Proc Symp. Pure Math 55 (1994), part 1, 163-187

[3] A, J, Scholl; Motives for modular forms; Inventiones math. 100 (1990), 419-430.

[4] J, Ayoub; Motives and algebraic cycles: a selection of conjectures and open questions. Preprint.

[5] J, Ayoub; Periods and the conjectures of Grothendieck and Kontsevich-Zagier. Newsletter of the European Mathematical Society, March 2014, Issue 91.
[6] Y, Andre; Galois theory, motives and transcendental numbers, arXiv:0805.2569v1 [math.NT]
[7] Y, Andre; Une Introduction aux Motifs (Motifs Purs, Motifs Mixtes, Priodes). Socit Mathmatique de France; distributed by the AMS
[8] S, Bloch, Algebraic cycles and higher K-theory, Advances in Mathematics 61 (3): 267 304
[9] V, Voevodsky, Triangulated categories of motives over a field, Cycles, Transfers, and Motivic Homology Theories, Princeton University Press,
[10] J. Ayoub, *L'Algèbre de Hopf et le groupe de Galois motiviques d'un corps de caractéristique nulle, II.*
[11] F, Brown, Motivic periods and P^1 minus 3 points (ICM proceedings) (2014), 1-25
[12] M, Nori, TIFR notes on motives, unpublished.
[13] M, Levine, *Mixed Motives*, K-theory Handbook.